8th Grade Algebra

2015-2016 Curriculum Map

Instruction and Course Alignment to the Common Core State Standards Content and Mathematical Practices

Course Unit Scope and Sequence

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Planning and Pacing Guide
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Mathematics
Curriculum and Instruction
Tacoma Public Schools

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Curriculum Map Unit Overview

	Unit	Description	Estimated Days
First Semester	Unit 1: Integer Exponents and Scientific Notation	**Grade 8 Module 1: Integer Exponents and Scientific Notation** This year begins with students extending the properties of exponents to integer exponents in Module 1. They use the number line model to support their understanding of the rational numbers and the number system. The number system is revisited at the end of the Semester (in Module 7) to develop the *real* number line through a detailed study of irrational numbers.	**10-15**
	Unit 2: Linear Equations	**Grade 8 Module 4: Linear Equations** In Module 4, students extend what they already know about unit rates and proportional relationships to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module. Students learn to apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality to transcribe and solve equations in one variable and then in two variables.	**35-40** (recommend finish by thanksgiving)
	Unit 3: Linear Functions	**Grade 8 Module 6: Linear Functions** In Grades 6 and 7, students worked with data involving a single variable. Module 6 introduces students to bivariate data. Students are introduced to a function as a rule that assigns exactly one value to each input. In this module, students use their understanding of functions to model the possible relationships of bivariate data. This module is important in setting a foundation for students' work in algebra second semester.	**15-20** (recommend finish by Winter Break)
	Unit 4: Introduction to Irrational Numbers using Geometry	**Grade 8 Module 7: Introduction to Irrational Numbers using Geometry** Module 7 begins with work related to the Pythagorean Theorem and right triangles, this should be a review of the concept taught in grade 7. In Module 7, lessons 1, 15, 16, 17, and 18 are review lessons. The majority of instructional time in this module should be spent on Topic B.	**10-15** (Recommend finish by Semester)

	Unit	Description	Estimated Days
Second Semester	Unit 5: Relationships Between Quantities and Reasoning with Equations and Their Graphs	**Algebra Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs** In this module students analyze and explain precisely the process of solving an equation. Through repeated reasoning, students develop fluency in writing, interpreting, and translating between various forms of linear equations and inequalities and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations. They learn the terminology specific to polynomials and understand that polynomials form a system analogous to the integers.	20
	Unit 6: Descriptive Statistics	**Algebra I Module 2: Descriptive Statistics** In this module, students reconnect with and deepen their understanding of statistics and probability concepts first introduced in Grades 6, 7, and 8. Students develop a set of tools for understanding and interpreting variability in data, and begin to make more informed decisions from data. They work with data distributions of various shapes, centers, and spreads. Students build on their experience with bivariate quantitative data from Grade 8. This module sets the stage for more extensive work with sampling and inference in later grades.	10
	Unit 7: Linear and Exponential Functions	**Algebra I Module 3: Linear and Exponential Functions** In earlier grades, students define, evaluate, and compare functions and use them to model relationships between quantities. In this module, students extend their study of functions to include function notation and the concepts of domain and range. They explore many examples of functions and their graphs, focusing on the contrast between linear and exponential functions. They interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations.	**25** (½ through unit by spring break)
	Unit 8: Polynomial and Quadratic Expressions, Equations, and Functions	**Algebra I Module 4: Polynomial and Quadratic Expressions, Equations, and Functions** In earlier modules, students analyze the process of solving equations and developing fluency in writing, interpreting, and translating between various forms of linear equations (Module 1) and linear and exponential functions (Module 3). These experiences combined with modeling with data (Module 2), set the stage for Module 4. Here students continue to interpret expressions, create equations, rewrite equations and functions in different but equivalent forms, and graph and interpret functions, but this time using polynomial functions, and more specifically quadratic functions, as well as square root and cube root functions.	**30**(start by May 1)

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Unit 1 – 8th Grade Module 1 Integer Exponents and Scientific Notation
Time Line 10 - 15 days

This year begins with students extending the properties of exponents to integer exponents in Module 1. They use the number line model to support their understanding of the rational numbers and the number system. The number system is revisited in Module 7 to develop the *real* number line through a detailed study of irrational numbers.

Big Ideas	Common Core State Standards	Standards for Mathematical Practice	Objectives	Engage Lessons	Assessments
Exponential Notation and Properties of Integer Exponents In this unit, students will explore patterns related to the properties of exponents as well as look for and make sense of the patterns they see. As students explore the properties of exponents they will make conjectures to support their mathematical claims regarding the properties. Accessing prior knowledge of the associative and distributive properties, students will apply them along with the properties of exponents. This unit begins with sense making focused on positive exponents and finishes with extending reasoning to negative and zero exponents.	**8.EE.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, 32 × 3-5 = 3-3 = 1/33 = 1/27.*	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. Students reason through the acceptability of definitions and proofs (e.g., the definitions of x0 and $x-b$ for all integers b and positive integers x). MP.6 Attend to precision. Beginning with the first lesson on exponential notation, students are required to attend to the definitions provided throughout the lessons and the limitations of symbolic statements, making sure to express what they mean clearly. MP.7 Look for and make use of structure. Students understand and make analogies to the distributive law as they develop properties of exponents. MP.8 Look for and express regularity in repeated reasoning.	• Students will use the mathematical practices to make conjectures about the properties of exponents. • Students know what it means for a number to be raised to a power and how to represent the repeated multiplication symbolically. • Students use the definition of exponential notation to make sense of the first law of exponents. • Students see a rule for simplifying exponential expressions involving division as a consequence of the first law of exponents. • Students write equivalent numerical and symbolic expressions using the first law of exponents. • Students will know how to take powers of powers. • Students will write simplified, equivalent numeric and symbolic expressions using this new knowledge of powers. • Students know that a number raised to the zeroth power is equal to one. • Students recognize the need for the definition to preserve the properties of exponents. • Students know the definition of a number raised to a negative exponent. • Students simplify and write equivalent expressions that contain negative exponents.	Topic A; Engage Lessons Number 1-5 (Lesson 6 can be omitted as proof of the properties of exponents is beyond first year algebra.)	

Magnitude and Scientific Notation Having established the properties of integer exponents, students learn to express the magnitude of a positive number through the use of scientific notation and to compare the relative size of two numbers written in scientific notation. Students explore uses of scientific notation and choose appropriately sized units as they represent, compare, and make calculations with very large quantities and very small quantities.	**8.EE.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3 × 108 and the population of the world as 7 × 109, and determine that the world population is more than 20 times larger.* **8.EE.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. Students reason through the acceptability of definitions and proofs (e.g., the definitions of x0 and x−b for all integers b and positive integers x). MP.6 Attend to precision. Beginning with the first lesson on exponential notation, students are required to attend to the definitions provided throughout the lessons and the limitations of symbolic statements, making sure to express what they mean clearly. MP.7 Look for and make use of structure. Students understand and make analogies to the distributive law as they develop properties of exponents. MP.8 Look for and express regularity in repeated reasoning.	• Students know positive powers of 10 are very large numbers, and negative powers of 10 are very small numbers. • Students know the exponent of an expression provides information about the magnitude of a number. • Students compare and estimate quantities in the form of a single digit times a power of 10. • Students simplify expressions using prior knowledge of ratios, fractions, and laws of exponents. • Students write and practice operations with numbers expressed in scientific notation and standard notation. • Students read, write, and perform operations on numbers expressed in scientific notation (very small and very large). Optional • Students understand how choice of unit determines how easy or difficult it is to understand an expression of measurement. • Students determine appropriate units for various measurements and rewrite measurements based on new units. • Students compare numbers expressed in scientific notation. Students apply the laws of exponents to interpret data and use technology to compute with very large numbers.	Topic B; Engage Lessons Number 7-13, with combining Lessons 8 and 11, and 12&13 optional.	

Unit 2 – Grade 8 Module 4 - Linear Equations **Time Line 35 - 40 days Recommend finishing by Thanksgiving**					
In Module 4, students extend what they already know about unit rates and proportional relationships (**6.RP.A.2**, **7.RP.A.2**) to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module (**8.EE.B.5**, **8.EE.B.6**). Also in this module, students learn to apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality (**6.EE.A.2**, **7.EE.A.1**, **7.EE.B.4**) to transcribe and solve equations in one variable and then in two variables.					
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Engage Lessons**	**Assessments**
Linear Equations and their Graphs Students begin by using their previous knowledge of multiple representations and the ideas of solutions to functions are an input and an output that makes an equation true. They dig in using a standard form linear equation. They find and organize solutions on a table and create a graph. As they explore these idea students begin to question whether their graph will always be a line as opposed to a curved shape. Continuing to use a standard form equation, students also explore the idea of vertical vs. horizontal lines on a coordinate plane. This initial look at linear functions will lead into the next section on slope and y-intercept.	8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	MP.1 **Make sense of problems and persevere in solving them.** Students analyze given constraints to make conjectures about the form and meaning of a solution to a given situation in one-variable and two-variable linear equations, as well as in simultaneous linear equations. MP.2 **Reason abstractly and quantitatively.** Students decontextualize and contextualize throughout the module as they represent situations symbolically and make sense of solutions within a context. MP.3 **Construct viable arguments and critique the reasoning of others.** Students use assumptions, definitions, and previously established facts throughout the module as they solve linear equations. Students make conjectures	Students use a table to find solutions to a given linear equation and plot the solutions on a coordinate plane. Students predict the shape of a graph of a linear equation by finding and plotting solutions on a coordinate plane. Students informally explain why the graph of a linear equation is not curved in terms of solutions to the given linear equation. Students graph linear equations in standard form, $ax + by = c$ (a or $b = 0$), that produce a horizontal or a vertical line.	Topic B Lessons 12, 13, 14	

Slopes and Equations of Lines Topic C begins with students examining the slope of non-vertical lines. They relate what they know about unit rate in terms of the slope of the graph of a line. Students reason that any two points on the same line can be used to determine slope because of what they know about similar triangles. They transform the standard form of an equation into slope-intercept form. Further, students learn that the slope of a line joining any two distinct points is the graph of a linear equation with slope, m. Students investigate the concept of uniqueness of a line and recognize that if two lines have the same slope and a common point; the two lines are the same. Students learn that the graph of a linear equation is a line and that a line is a graph of a linear equation and will focus on the graphical location of y-intercept. They will write the equation of a line given the slope and a point. Students learn that any constant rate problem can be described by a linear equation in two variables where the slope of the graph is the constant rate, i.e., rate of change. Given a graph and an equation, students must use what they know about slope to determine which of the two has a greater rate of change. Students must know that different forms (standard, slope-intercept, point-slope) of linear equations may appear to be different but in actuality they can be the same graph and represent the same line.	8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. 8.EE.B.6 Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ of a line intercepting the vertical axis at *b*.	about the graph of a linear equation being a line, then proceed to prove this claim. **MP.4 Model with mathematics.** Throughout the module, students represent real-world situations symbolically. **MP.7 Look for and make use of structure.** Students use the structure of an equation to make sense of the information in the equation.	Students know slope is a number that describes the steepness or slant of a line. Students interpret the unit rate as the slope of a graph. Students use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. Students use the slope formula to compute the slope of a non-vertical line. Students show that the slope of a line joining any two distinct points of the graph of $y = mx+b$ has slope, m. Students transform the standard form of an equation into $y = -(A/B)x+C/B$. Students graph equations in the form of $y=mx+b$ using information about slope and y-intercept. Students know that if they have two straight lines with the same slope and a common point that the lines are the same. Students graph linear equations on the coordinate plane. Students write the equation that represents the graph of a line. Students write the equation of a line given two points or the slope and a point on the line. Students know the traditional forms of the slope formula and slope-intercept equation. Students know that any constant rate problem can be described by a linear equation in two variables where the slope of the graph is the constant rate. Students compare two different proportional relationships represented by graphs, equations, and tables to determine which has a greater rate of change. Students know that Ax + By = C can be transformed into an infinite number of equivalent equations by use of a scale factor; equivalent equations result in the same graph.	Topic C Lessons 15-23 Notes: Students are not expected to know the proofs presented in Lessons 19 and 20. Lesson 19&20 can be combined into one day. Lesson 21 can be supplemented with the point-slope form of a line.	

Systems of Linear Equations and their Solutions This topic introduces students to systems of linear equations with a comparison of distance-time graphs in order to determine which of two objects has greater speed. From there, students graph two linear equations and identify their point of intersection as the solution to the system. Next, students look at systems of equations that graph as parallel lines. Students learn that a system can have no solution because parallel lines do not have a point of intersection. This thinking continues with respect to systems that have infinitely many solutions. Graphic representations and solutions are then shifted to learning how to solve a system of equations using computational methods like elimination and substitution.	8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. 8.EE.C.8 Analyze and solve pairs of simultaneous linear equations: a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*		Students compare the graphs that comprise a system of linear equations, in the context of constant rates, to answer questions about time and distance. Students graph two equations and find the point of intersection and identify the point of intersection of the two lines as the solution to the system. Students verify by computation that the point of intersection is a solution to each of the equations in the system. Students know that when a system of linear equations has no solution, i.e., no point of intersection of the lines, then the lines are parallel. Students know that equivalent equations have infinitely many solutions. Students learn, know and demonstrate a strategy for solving a system of linear equations algebraically – elimination and substitution. Students use properties of rational numbers to find a solution to a system, if it exists, through computation using substitution and elimination methods. Students write word problems into systems of linear equations.	Topic D Lessons 24-29 Lesson 30 can be used as the Anchor Task for this Unit	

Unit 3 – 8th Grade Module 6 – Linear Functions
Time Line 15 - 20 days   Recommend finishing by Winter Break

In Grades 6 and 7, students worked with data involving a single variable. Module 6 introduces students to bivariate data. Students are introduced to a function as a rule that assigns exactly one value to each input. In this module, students use their understanding of functions to model the possible relationships of bivariate data. This module is important in setting a foundation for students' work in algebra in Grade 9.

Big Ideas	Common Core State Standards	Standards for Mathematical Practice	Objectives	Engage Lessons	Assessments
Linear Functions In Topic A, students build on their study of functions by recognizing a linear relationship between two variables (**8.F.B.4**). Students use the context of a problem to construct a function to model a linear relationship (**8.F.B.4**). In Lesson 1, students are given a verbal description of a linear relationship between two variables and then must describe a linear model. Students graph linear functions using a table of values and by plotting points. They recognize a linear function given in terms of the slope and initial value, or yy-intercept. In Lesson 2, students interpret the rate of change and the yy-intercept, or initial value, in the context of the problem. They interpret the sign of the rate of change as indicating that a linear function is increasing or decreasing (**8.F.B.5**) and as indicating the steepness of a line. In Lesson 3, students graph the line of a given linear function. They express the equation of a linear function as $yy=mmxx+bb$, or an equivalent form, when given the initial value and slope. In Lessons 4 and 5, students describe and interpret a linear function given two points or its graph.	8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(xx,)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	MP.2 **Reason abstractly and quantitatively.** Students reason quantitatively by symbolically representing the verbal description of a relationship between two bivariate variables. They attend to the meaning of data based on the context of problems and the possible linear or nonlinear functions that explain the relationships of the variables. MP.4 **Model with mathematics.** Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data. MP.6 **Attend to precision.** Students evaluate functions to model a relationship between numerical variables. They evaluate the function by assessing the closeness of the data points to the line. They use care in	• Students determine a linear function given a verbal description of a linear relationship between two quantities. • Students interpret linear functions based on the context of a problem. • Students sketch the graph of a linear function by constructing a table of values, plotting points, and connecting points by a line. • Students interpret the constant rate of change and initial value of a line in context. • Students interpret slope as rate of change and relate slope to the steepness of a line and the sign of the slope, indicating that a linear function is increasing if the slope is positive and decreasing if the slope is negative. • Students graph a line specified by a linear function. • Students graph a line specified by an initial value and rate of change of a function and construct the linear function by interpreting the graph. • Students graph a line specified by two points of a linear relationship and provide the linear function. • Students describe qualitatively the functional relationship between two types of quantities by analyzing a graph. • Students sketch a graph that exhibits the qualitative features of a function based on a verbal description. • Students qualitatively describe the functional relationship between two types of quantities by analyzing a graph. • Students sketch a graph that exhibits the qualitative features of linear and nonlinear functions based on a verbal description.	Topic A Lessons 1 through 5	

		interpreting the slope and the yy-intercept in linear functions. **MP.7 Look for and make use of structure.** Students identify pattern or structure in scatter plots. They fit lines to data displayed in a scatter plot and determine the equations of lines based on points or the slope and initial value.			
Bivariate Numerical Data In Topic B, students connect their study of linear functions to applications involving bivariate data. A key tool in developing this connection is a scatter plot. In Lesson 6, students construct scatter plots and focus on identifying linear versus nonlinear patterns (**8.SP.A.1**). They distinguish positive linear association and negative linear association based on the scatter plot. Students describe trends in the scatter plot along with clusters and outliers (points that do not fit the pattern). In Lesson 8, students informally fit a straight line to data displayed in a scatter plot (**8.SP.A.2**) by judging the closeness of the data points to the line. In Lesson 9, students interpret and determine the equation of the line they fit to the data and use the equation to make predictions and to evaluate possible association of the variables. Based on these predictions, students address the need for a *best-fit* line, which is formally introduced in Algebra I.	**8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.		• Students construct scatter plots. • Students use scatter plots to investigate relationships. • Students understand that a trend in a scatterplot does not establish cause-and-effect. • Students distinguish linear patterns from nonlinear patterns based on scatter plots. • Students describe positive and negative trends in a scatter plot. • Students identify and describe unusual features in scatter plots, such as clusters and outliers. • Students informally fit a straight line to data displayed in a scatter plot. • Students make predictions based on the graph of a line that has been fit to data.	Topic B Lessons 6 through 9	

Linear and Nonlinear Models In Topic C, students interpret and use linear models. They provide verbal descriptions based on how one variable changes as the other variable changes (**8.SP.A.3**). Students identify and describe how one variable changes as the other variable changes for linear and nonlinear associations. They describe patterns of positive and negative associations using scatter plots (**8.SP.A.1**, **8.SP.A.2**). In Lesson 10, students identify applications in which a linear function models the relationship between two numerical variables. In Lesson 11, students use a linear model to answer questions about the relationship between two numerical variables by interpreting the context of a data set (**8.SP.A.1**). Students use graphs and the patterns of linear association to answer questions about the relationship of the data. In Lesson 12, students also examine patterns and graphs that describe nonlinear associations of data (**8.SP.A.1**).	**8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of* 1.5 cm/hr *as meaning that an additional hour of sunlight each day is associated with an additional* 1.5 cm *in mature plant height.*		• Students identify situations where it is reasonable to use a linear function to model the relationship between two numerical variables. • Students interpret slope and the initial value in a data context.	Topic C Lesson 10, 11, and 12	

Bivariate Categorical Data Topic D extends the concept of a relationship between variables to bivariate categorical data. In Lesson 13, students organize bivariate categorical data into a two-way table (**8.SP.A.4**). They calculate row and column relative frequencies and interpret them in the context of a problem. They informally decide if there is an association between two categorical variables by examining the differences of row or column relative frequencies. They interpret association between two categorical variables as knowing the value of one of the variables provides information about the likelihood of the different possible values of the other variable.	**8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*		• Students organize bivariate categorical data into a two-way table. • Students calculate row and column relative frequencies and interpret them in context. • Students use row relative frequencies or column relative frequencies to informally determine whether there is an association between two categorical variables.	Topic D Lesson 13 and 14	

Unit 4 – 8th Grade Module 7 – Introduction to Irrational Numbers using Geometry Time Line 10 - 15 days Recommend finishing by Semester Break					
Students extend what they already know about unit rates and proportional relationships to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module. Students learn to apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality to transcribe and solve equations in one variable and then in two variables					
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Engage Lessons**	**Assessments**
Square and Cube Roots In Topic A, students are introduced to the notation and meaning of square roots. They understand that most square roots are not perfect squares, and estimate their position on a number line between integers. Students determine that most square roots lie between integers on a number line, and use integers to estimate their position. Students will find the square root or cube root of a number using an equation such as $x^2 = p$, where p is a positive rational number. Students will also simplify square roots as a product of its factors. i.e., $\sqrt{18} = 3\sqrt{2}$.	**8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi 2$). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between* 1 *and* 2*, then between* 1.4 *and* 1.5*, and explain how to continue on to get a better approximation.* **8.EE.A.2** Use square root and cube root symbols to represent solutions to the equations of the form $x2 = p$ and $x3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**MP.6 Attend to precision**. Students begin attending to precision by recognizing and identifying numbers as rational or irrational. **MP.7 Look for and make use of structure**. Students learn that a radicand can be rewritten as a product and that sometimes one or more of the factors of the product can be simplified to a rational number. **MP.8 Look for and express regularity in repeated reasoning**. While using the long division algorithm to convert fractions to decimals, students recognize that when a sequence of remainders repeats that the decimal form of the number will contain a repeat block.	• Students find the square root of small perfect squares and estimate square roots of non-perfect squares. • Students approximate the location of square roots on the number line. • Students know that the positive square root and cube root exists for all positive numbers and is unique. • Students solve simple equations that require them to find the square or cube root of a number. • Students use factors of a number to simplify a square root.	Topic A Lessons 2, 3, and 4	

Decimal Expansions of Numbers In Topic B students will distinguish between irrational and non-terminating rational numbers. They will generalize the conditions for when a number has a finite decimal expansion, and when it contains a repeating block of digits. They will understand that all other numbers are irrational.			• Students know that every number has a finite or infinite decimal expansion, and that fractions with a denominator that is the product of 2's and/or 5's have finite decimal expansions. • Students know the meaning of an infinite decimal. • Students will be able to explain why the infinite decimal 0.9… is equal to 1. • Students use the long division algorithm to derive the division-with-remainder or the decimal expansion of a number. • Students know why digits repeat in terms of the algorithm. • Students know that every rational number has a decimal expansion that repeats eventually. • Students apply knowledge of equivalent fractions, long division, and the distributive property to write the decimal expansion of fractions. • Students know why every repeating decimal is equal to a fraction, and that the decimal expansions of rational numbers repeat eventually. • Students convert a decimal expansion that eventually repeats into a fraction. • Students know the meaning of irrational numbers, and that irrational numbers cannot be represented as a fraction, and have infinite decimals that never repeat. • Students distinguish between rational and irrational numbers based on decimal expansions, and use rational approximation to get the approximate decimal expansion of irrational numbers. • Students determine the decimal expansion of a fraction. • Students relate the method of rational approximation to the long division algorithm. • Students use rational approximations of irrational numbers to compare the size of irrational numbers, and to place them	Topic B Lessons 6,7,8,9,10,11, 12 and 13 Lesson 14 is optional	

			in their approximate locations on a number line.		
The Pythagorean Theorem In Topic C students build on knowledge of the Pythagorean Theorem from 7th grade and understand that side lengths of right triangles are often expressed as irrational numbers. They develop understanding of the Pythagorean Theorem and its converse through various proofs. Students apply the Pythagorean Theorem to find distance between points on a coordinate grid, and use it in practical applications.	8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse. 8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	MP.6 **Attend to precision**. Students begin attending to precision by recognizing and identifying numbers as rational or irrational. MP.7 **Look for and make use of structure**. Students learn that a radicand can be rewritten as a product and that sometimes one or more of the factors of the product can be simplified to a rational number. MP.8 **Look for and express regularity in repeated reasoning**. While using the long division algorithm to convert fractions to decimals, students recognize that when a sequence of remainders repeats that the decimal form of the number will contain a repeat block.	Students know that when the square of a side of a right triangle represented as a^2, b^2, or c^2 is not a perfect square, they can estimate the side length as between two integers and identify the integer to which the length is closest. Students know that the Pythagorean Theorem can be interpreted as a statement about the areas of similar geometric figures constructed on the sides of a right triangle. Students explain a proof of the Pythagorean Theorem. Students explain a proof of the converse of the Pythagorean Theorem. Students apply the theorem and its converse to solve problems. Students determine the distance between two points on a coordinate plane using the Pythagorean Theorem. Students apply the Pythagorean Theorem to real world and mathematical problems in two dimensions.	Pythagorean Theorem was introduced in Grade 7 using Engage NY Grade 8 Module 2 and 3. Topic C Lessons 1, 15-18 Supplemental Lesson can come from Grade 8 Modules 2 and 3 based on student's prior knowledge.	

Unit 5 – Algebra Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs Time Line 20 days Recommend finishing by the end of February					
In eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations. They learn the terminology specific to polynomials and understand that polynomials form a system analogous to the integers.					
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Engage Lessons**	**Assessments**
Introduction to Functions Studied This Year—Graphing Stories Topic A, students explore the main functions that they will work with in algebra: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of real world situations. As they graph, they reason abstractly and quantitatively as they choose and interpret units to solve problems related to the graphs they create.	**N-Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2** Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★	**MP.1 Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem. **MP.2 Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities. **MP.3 Construct viable arguments and critique the reasoning of others.** Students reason about solving equations using “if-then” moves based on equivalent expressions and properties of equality and inequality. They analyze when an “if-then” move is not reversible.	Students define appropriate quantities from a situation and can choose a scale that fits the situation for a piecewise linear graph. Students represent graphically and interpret a non-linear relationship between two quantities. Students choose and interpret the scale on a graph to appropriately represent an exponential function. Students develop the tools necessary to discern units for quantities in real-world situations and choose levels of accuracy appropriate to limitations on measurement.	Topic A Lessons 1,2,3,4 Lesson 5 will be skipped because it references systems	

The Structure of Expressions The goal of this topic is to address a fundamental, underlying question: *Why are the commutative, associative, and distributive properties so important in mathematics?*1 The answer to the question is, of course, because these three properties help to generate all equivalent algebraic expressions discussed in Algebra I. Students will apply the properties to see that they can create equivalent expressions through the repeated use of these properties. Students continue practicing applying properties of simplification by combining like terms when adding and subtracting polynomials. They also get a chance to compare what multiplying polynomials does differently. They explore the idea that adding, subtracting and multiplying polynomials produces another polynomial.	A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	MP.4 **Model with mathematics.** Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system. MP.6 **Attend to precision.** Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation. MP.7 **Look for and make use of structure.** Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc. MP.8 **Look for and express regularity in repeated reasoning.** After solving many linear equations in one	Students use the structure of an expression to identify ways to rewrite it. Students use the distributive property to prove equivalency of expressions. Students use the commutative and associative properties to recognize structure within expressions and to prove equivalency of expressions. Students understand that the sum or difference of two polynomials produces another polynomial and relate polynomials to the system of integers; students add and subtract polynomials. Students understand that the product of two polynomials produces another polynomial; students multiply polynomials.	Topic B Lessons 6-9	
Solving Equations and Inequalities This is THE solve unit. We are assuming that students are coming in from 7th grade with a general knowledge of solving equations and inequalities. This should include solving with x on	A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities*		Understand that an equation is a statement of equality between two expressions. When values are substituted into an equation the equation is either true or false. A solution is a value that makes an equation true. Make connections that an equation is a filter that "sifts" through the domain of the	Topic C Lessons 10-21	

both sides. (Please note you may need some supplemental practice here). Our goal in algebra is for students to make sense of solving all kinds of equations and inequalities (listed in objectives). Students should be given multiple opportunities and multiple strategies/connections that allow them to make sense out of the solve concept. Students should make connections to what "IS a SOLUTION" vs. what "is NOT a SOLUTION". An equation with variables can be viewed as a question asking for which values of the variables (the solution set) will result in true number sentences when those values are substituted into the equation. Students should make connections…NOT just follow a set of "rules". In this unit, we explore linear and non-linear ideas that will reappear throughout the course.	*describing nutritional and cost constraints on combinations of different foods*. A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.* A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	variable (e.g., $3x + 5=8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b = cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.	values to find the "solution set" and the numbers for which the equation is false. Students understand the commutative, associate, and distributive properties as identities. Students are introduced to the formal process of solving an equation: starting from the assumption that the original equation has a solution. Students explain each step as following from the properties of equality. Students identify equations that have the same solution set (one solution, no solutions or infinitely many solutions) Students learn and practice "if-then" moves using the properties of equality to solve equations and inequalities. Types of equations include: *2 step one variable equations *Multi Step one variable equations *1 variable inequalities *2 variable inequalities *2 equations joined by "and" or "or" *2 inequalities joined by "and" or "or" *A factored quadratic equation = to 0 *Equations with variable expression in denominator *Rearranging formulas to solve for any unknown variable Students explore and begin to recognize solution sets on a number line and coordinate plane using inequalities such as x > 5 and 2x + 4y < 12. Students understand that a half-plane bounded by the line $ax + by = c$ is a visual representation of the solution set to a linear inequality such as $ax + by < c$. They interpret the inequality symbol correctly to determine which portion of the		

	A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		coordinate plane is shaded to represent the solution.		
Creating Equations to Solve Problems As a summary to this unit, students are introduced to the modeling cycle (see page 61 of the Common Core Learning Standards) through problems that	N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. A-SSE.1 Interpret expressions that represent a quantity in		Students investigate a problem that can be solved by reasoning quantitatively and by creating equations in one variable. They compare the numerical approach to the algebraic approach. Students learn the meaning and notation of recursive sequences in a modeling setting.	Topic D Lesson 25 Lessons 26 & 27 are application lessons and have good problems that could be integrated into earlier lesson.	

can be solved using equations and inequalities in one variable and graphing. Modeling links classroom mathematics and statistics to everyday life, work, and decision-making.	terms of its context. A-SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients A-SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.		Students use recursive sequences to model and answer problems. Students create equations and inequalities to solve a modeling problem. Students represent constraints by equations and inequalities and interpret solutions as viable or non-viable options in a modeling context.		

Unit 6 – Algebra Module 2 Descriptive Statistics
Time Line 10 days

Module 2: This module builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students display and interpret graphical representations of data, and if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities.

Big Ideas	Common Core State Standards	Standards for Mathematical Practice	Objectives	Engage Lessons	Assessments
Shapes and Centers of Distributions In topic A, students observe, describe and deepen their understanding of data distributions. They should recognize that the values of mean and median are different for skewed distributions and similar for symmetrical distributions. Students use the balance point to select a measure of center based on the distribution shape to appropriately describe a typical value for the data distribution.	S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	MP.1 **Make sense of problems and persevere in solving them.** Students choose an appropriate method of analysis based on problem context. MP.2 **Reason abstractly and quantitatively.** Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. MP.3 **Construct viable arguments and critique the reasoning of others.** Students examine the shape, center, and variability of a data distribution and use characteristics of the data distribution to communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students. MP.4 **Model with mathematics.** Students construct and interpret two-way tables to summarize bivariate categorical data.	Students use informal language to describe the shape, center, and variability of a distribution based on a dot plot, histogram, or box plot. Students make meaningful conjectures to connect data distributions to their contexts and the questions that could be answered by studying the distributions. Students construct a dot plot from a data set. Students calculate the mean of a data set and the median of a data set. Students observe and describe that measures of center (mean and median) are nearly the same for distributions that are nearly symmetrical. Students observe and explain why the mean and median are different for distributions that are skewed. Students select the mean as an appropriate description of center for a symmetrical distribution and the median as a better description of center for a distribution that is skewed. Students estimate the mean and median of a distribution represented by a dot plot or a histogram. Students indicate that the mean is a reasonable description of a typical value for a distribution that is symmetrical but that the median is a better description of a typical value for a distribution that is skewed. Students interpret the mean as a balance point of a distribution.	Topic A Lessons 1-3	

		MP.5 Use appropriate tools strategically. Students visualize data distributions and relationships between numerical variables using graphing software.	Students indicate that for a distribution in which neither the mean nor the median is a good description of a typical value, the mean still provides a description of the center of a distribution in terms of the balance point.		
Describing Variability and Comparing Distributions In Topic B, students deepen understanding of measures of variability by connecting a measure of the center of a data distribution to an appropriate measure of variability. Students calculate and interpret the mean absolute deviation and the standard deviation to describe variability for data distributions that are approximately symmetric. Students also look at box plots for skewed data to select a measure of center and variability around that center.	S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**MP.6 Attend to precision.** Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students use statistical terminology appropriately.	Students calculate the deviations from the mean for two symmetrical data sets that have the same means. Students interpret deviations that are generally larger as identifying distributions that have a greater spread or variability than a distribution in which the deviations are generally smaller. Students calculate the standard deviation for a set of data. Students interpret the standard deviation as a typical distance from the mean. Students compare the relative variability of distributions using standard deviations. Students explain why a median is a better description of a typical value for a skewed distribution. Students construct a box plot based on the 5-number summary and calculate the interquartile range (IQR). Students interpret the IQR as a description of variability in the data. Students identify outliers in a data distribution. Students compare two or more distributions in terms of center, variability, and shape. Students interpret a measure of center as a typical value. Students answer questions that address differences and similarities for two or more distributions.	Topic B Lessons 4-8	
Categorical Data on Two Variables In Topic C, students work with categorical data to analyze and draw conclusions. Students will organize the data using two-way frequency tables to calculate the conditional relative frequencies. They will explore a possible association between two	S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations		Students distinguish between categorical data and numerical data. Students summarize data on two categorical variables collected from a sample using a two-way frequency table. Given a two-way frequency table, students construct a relative frequency table and interpret relative frequencies.	Topic C Lessons 9-11 Lessons are optional depending on time available to finish first semester.	

categorical variables using differences in conditional and relative frequencies. Teachers should emphasize critically thinking about association between two categorical variables and a causal relationship between two variables (association does not imply causation). This provides a foundation for work on sampling and inference in later grades.	and trends in the data. S-ID.9 Distinguish between correlation and causation.		Students calculate and interpret conditional relative frequencies from two-way frequency tables. Students evaluate conditional relative frequencies as an indication of possible association between two variables. Students explain why association does not imply causation.		
Numerical Data on Two Variables In Topic D, students analyze relationships between two quantitative variables using scatterplots and by summarizing linear relationships using the least squares regression line. Models are proposed based on an understanding of the equations representing the models and the observed pattern in the scatter plot. Students calculate and analyze residuals based on an interpretation of residuals as prediction errors. Teachers may wish to use lesson 20 as an anchor activity.	S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit the function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	MP5 **Use appropriate tools strategically.** Students visualize data distributions and relationships between numerical variables using graphing software. They select and analyze models that are fit using appropriate technology to determine whether or not the model is appropriate. Students use visual representations of data distributions from technology to answer statistical questions. MP.6 **Attend to precision.** Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students use statistical terminology appropriately.	Students distinguish between scatter plots that display a relationship that can be modeled by a linear equation vs. a nonlinear equation. Students use an equation given as a model for a nonlinear relationship to answer questions based on the equation and the context of the data. Students determine the least-squares regression line from a given set of data using technology and use it to predict values for a given data set. * Students use residuals to evaluate the accuracy of predictions based on the least-squares line. * Students use a graphing calculator to construct the residual plot for a given data set. * Students use a residual plot as an indication of whether the model used to describe the relationship between two numerical variables is an appropriate choice. * Students use technology to determine the value of the correlation coefficient for a given data set.	Topic D Engage Lessons 12-20	

	S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. S-ID.9 Distinguish between correlation and causation.		Students interpret the value of the correlation coefficient as a measure of strength and direction of a linear relationship. Students explain why correlation does not imply causation. Students construct a scatter plot of given data. Students analyze their data, examining the residual plot, and interpreting the correlation coefficient. (* means that the outcome/objective is repeated throughout the unit)		

Unit 7 – Algebra Module 3 – Linear and Exponential Functions Time Line 25 days					
Module 3: In earlier grades, students defined, evaluated, and compared functions in modeling relationships between quantities. In this module, students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on their understanding of integer exponents to consider exponential functions with integer domains. They compare and contrast linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. In building models of relationships between two quantities, students analyze the key features of a graph or table of a function.					
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Engage Lessons**	**Assessments**
Linear and Exponential Functions In Topic A, students explore arithmetic and geometric sequences as an introduction to the formal notation of functions. They interpret arithmetic sequences as linear functions with integer domains and geometric sequences as exponential functions with integer domains. Students compare and contrast the rates of change of linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. They identify structures of exponential functions that have a growth scenario vs. a decay scenario and can work with both.	F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation *y=f(x)*. F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F-IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n–1) for n ≥ 1.* F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	**MP.1 Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain insight into the problem. **MP.2 Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements	Students examine sequences and are introduced to the notation used to describe them. Students write sequences with recursive and explicit formulas. Students learn the structure of arithmetic and geometric sequences. Students compare the rate of change for simple and compound interest and recognize situations in which a quantity grows by a constant percent rate per unit interval. Students are able to model and solve problems involving exponential functions. Students compare linear and exponential models of population growth.	Topic A Engage Lesson 1-7	

	F-BF.A.1a Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★ a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. F-LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	and apply reason (from the shape of the graphs) to infer the quantities being displayed and consider possible units to represent those quantities. MP.4 **Model with mathematics.** Students have numerous opportunities to solve problems that arise in everyday life, society, and the workplace (e.g., modeling bacteria growth and understanding the federal progressive income tax system). MP.7 **Look for and make use of structure.** Students reason with and analyze collections of equivalent expressions to see how they are linked through the properties of	Students describe and analyze exponential growth and decay models.		
Functions and Their Graphs In Topic B, students connect their understanding of functions to their knowledge of graphing. They learn the formal definition of a function and how to recognize, evaluate, and interpret functions in abstract and contextual situations. Students examine the graphs of a variety of functions and learn to interpret those graphs using precise terminology to describe such key features as domain and range, intercepts, intervals where the function is increasing or decreasing, and intervals where the function is positive or negative. Students understand that functions can be linear or non linear.	F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then (x) denotes the output of *f* corresponding to the input x. The graph of f is the graph of the equation y=(x). F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. F-IF.B.4 For a function that models a relationship between		Students use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Students create functions that represent a geometric situation and relate the domain of a function to its graph and to the relationship it describes. Students understand that a function from one set (called the domain) to another set (called the range)	Topic B Engage Lessons 8-10, 13, 14 It is important to have supports for students understanding of domain and range throughout this unit. Lesson 13 should focus on sense making around increasing and decreasing intervals. References to set builder notation	

	two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* F-IF.C.7a Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves. (e.g., $2x+4=10$, $2(x-3)+4=10$, $2(3x-4)+4=10$) MP.8 **Look for and express regularity in repeated reasoning.** After solving many linear equations in one variable (e.g., $3x+5=8x-17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters $(e.g.,+b=cx+d)$. They pay close attention to calculations involving the properties of operations,	assigns each element of the domain to exactly one element of the range. Students understand that if f is a function and x is an element of its domain, then (x) denotes the output of f corresponding to the input x. Students create tables and graphs of functions and interpret key features including intercepts, increasing and decreasing intervals, and positive and negative intervals. Students compare linear and exponential models by focusing on how the models change over intervals of equal length. Students observe from tables that a function that grows exponentially will eventually exceed a function that grows linearly.	and pseudo code do not apply to first year algebra.	
Transformations of Functions In Topic C, students extend their understanding of piecewise, absolute value, and step functions. They learn that a graphical approach to finding a solution to any	A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using		Students examine the features of piecewise functions including the absolute value function and step functions. Students understand that the graph of a function f is the graph of the equation $y=(x)$. Students discover that the multi-step and exact way of solving $\|2x-5\|=\|3x+1\|$ using algebra can sometimes be avoided by recognizing that an equation of the form $(x)=g(x)$ can be solved visually by looking for	Topic C Lessons 15-20 Transformations need to focus on vertical, horizontal, and vertical dilations, and connected to work on transformations in geometry units.	

system of functions is the intersection of the two graphs. For example, students may explore the intersection of a linear function and a quadratic function. Students investigate transformations of functions and draw formal conclusions about the effects of a transformation on the function's graph.	technology to graph the functions, make tables of values, or find successive approximations. Include cases where (x) and/or (x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. F-IF.C.7a Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. F-BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their*	properties of equality, and properties of inequalities, to find equivalent expressions and solve equations, while recognizing common ways to solve different types of equations.	the intersection points of the graphs of $y=f(x)$ and $y=g(x)$. Students examine that a vertical translation of the graph of $y=(x)$ corresponds to changing the equation from $y=f(x)$ to $y=f(x)+k$. Students examine that a vertical scaling of the graph of $y=(x)$ corresponds to changing the equation from $y=f(x)$ to $y=kf(x)$. Students examine that a horizontal translation of the graph of $y=(x)$ corresponds to changing the equation from $y=f(x)$ to $y=f(x-k)$. Students apply their understanding of transformations of functions and their graphs to piecewise functions.		

	graphs and algebraic expressions for them.				
Using Functions and Graphs to Solve Problems In topic D students apply and reinforce the concepts of this module as they examine and compare all types of functions in real-world contexts. They will do this by creating functions to model situations, relating elements of a representation of a function to the context of the problem.	A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A-SSE.B.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.⋆ c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is* 15%. F-FI.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in		Students create models and understand the differences between linear and exponential models that are represented in different ways. Students apply knowledge of exponential functions and transformations of functions to a contextual situation. Students apply knowledge of exponential functions and transformations of functions to a contextual situation.	Topic D Lesson 21 and 22 only	

	terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*. F-IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F-IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger*				

	maximum. F-BF.A.1a Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context.				

Unit 8 – Algebra Module 4 – Polynomial and Quadratic Expressions, Equations, and Functions Time Line 30 days Recommend starting by May 1					
Module 4: In this module, students build on their knowledge from Module 3. Students strengthen their ability to discern structure in polynomial expressions. They create and solve equations involving quadratic and cubic expressions. In this module's modeling applications, students reason abstractly and quantitatively in interpreting parts of an expression that represent a quantity in terms of its context; they also learn to make sense of problems and persevere in solving them by choosing or producing equivalent forms of an expression (e.g., completing the square in a quadratic expression to reveal a maximum value). Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They learn through repeated reasoning to anticipate the graph of a quadratic function by interpreting the structure of various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function.					
Big Ideas	**Common Core State Standards**	**Standards for Mathematical Practice**	**Objectives**	**Engage Lessons**	**Assessments**

Quadratic Expressions, Equations, Functions, and Their Connection to Rectangles Students will extend the work they did in unit 1 with polynomials to multiplying and factoring polynomials, including area models. The zero property of multiplication will be introduced, and students will solve factored expressions. They will translate between standard and factored forms, and make connections between the roots, zeros, x-intercepts, vertex and line of symmetry of a parabola. Students will compare and contrast the domain and range of the algebraic equations and how these may need to be restricted for situations and contexts.	**A-SSE.A.1** Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* **A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* **A-SSE.B.3a** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A) Factor a quadratic expression to reveal the zeros of the function it defines. **A-APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **A-REI.B.4b** Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. [Note: Tasks do not require students to write	**MP.1 Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. **MP.2 Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpretation of the quantities in the relationship in terms of the context. Students must be able to *decontextualize*—to abstract a given situation and represent it	Students use the distributive property and area models to multiply polynomials. Students generate conjectures for the square of a binomial and difference of squares. Students understand that factoring reverses the multiplication process. Students factor trinomials with a leading co-efficient a=1 and a not equal to one. Students factor quadratic expressions that cannot be easily factored and develop additional strategies for factorization, including splitting the linear term, using graphing calculators, and using geometric or tabular models. Students solve increasingly complex one-variable equations, some of which need algebraic manipulation, including factoring as a first step and using the zero product property. Students use appropriate and efficient strategies to find solutions to basic quadratic equations. Students interpret verbal and word problems to create equations in one-variable and solve them (i.e., determine the solution set) using factoring and the *zero product property*.	Topic A Engage Lessons 1-10	

	solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions. **A-REI.D.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f(x)* and *y* = *g(x)* intersect are the solutions of the equation *f(x)* = *g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. **A-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. **A-CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.	symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents, and then to *contextualize*—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They use multiple representations modeling situations. **MP.5 Use appropriate tools strategically.** Mathematically	Students examine quadratic equations in two variables represented graphically on a coordinate plane and recognize the symmetry of the graph. They explore key features of graphs of quadratic functions: y-intercept and x-intercepts, the vertex, the axis of symmetry, increasing and decreasing intervals, negative and positive intervals, and end behavior. They sketch graphs of quadratic functions as a symmetric curve with a highest or lowest point corresponding to its vertex and an axis of symmetry passing through the vertex. Students can go between different representations of a quadratic function, and use the relationship between the roots, vertex and line of symmetry. Students understand the relationship between the degree of a polynomial and the number of roots and maxima and minima Students interpret quadratic functions from graphs and tables: intercepts, vertex, axis of symmetry, positive and negative values for the function, increasing and decreasing intervals, and the graph's end behavior. Students determine an appropriate domain and range for a function's graph and when given a context.		

	F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **F-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.B.7a** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	proficient students consider the available tools when solving a mathematical problem. **MP.6 Attend to precision.** Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. **MP.7 Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. In this Module, students use the structure of expressions to find ways to rewrite them in different but equivalent forms.			
Using Different Forms for Quadratic Functions Students extend topic A, and complete the square to reveal the vertex form of a quadratic equation, and understand the three forms of the quadratic equation reveal different properties about the graph.	**N-RN.B.3**Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **A-SSE.A.1**Interpret expressions that represent a quantity in		Students will translate between the standard form of a quadratic and the vertex form through completing the square. Students solve complex quadratic equations, including those with a leading coefficient other than 1, by completing the square.	Topic B Engage Lessons 11-17	

Students will derive the quadratic formula, and use the discriminate to determine the number of solutions. For quadratics without rational solutions, students will make connections to irrational numbers from 1st semester. Students will use quadratics to model situations from business and physics.	terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.* **A-SSE.A.2**Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).* **A-SSE.B.3a,b**Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **A-APR.B.3**Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **A-CED.A.1**Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **A-CED.A.2**Create equations in two or more variables to represent relationships between quantities; graph equations on		Students derive the quadratic formula by completing the square for a general quadratic equation in standard form. Students use the quadratic formula to solve quadratic equations that cannot be easily factored. Students understand that the discriminant can be used to determine the number of solutions/roots/intercepts. Students graph simple quadratic equations of the form $y = a(x - h)^2 + k$ Students understand the relationship between the leading coefficient and the shape of the parabola. Students graph a variety of quadratic functions using the form $(x)=ax^2+bx+c$ (standard form). Students analyze and draw conclusions about contextual applications using the key features of a function and its graph.		

	coordinate axes with labels and scales. **A-REI.B.4**Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form *(x – p) 2 = q* that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x*2* = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. **F-IF.B.4**For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* **F-IF.B.6**Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **F-IF.C.7a**Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★				

	a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **F-IF.C.8a**Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.				
Function Transformations and Modeling Students will combine their knowledge of quadratics and transformation and extend these ideas to new parent functions. Students will understand how constants will dilate the graph. Students will apply these models to solve problems from business and physics.	A-CED.A.2Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. F-IF.6Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. F-IF.C.7bGraph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. F-IF.C.9Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*		Students can graph and understand the properties of the x^2, x^3 and $x^{(1/2)}$ parent functions. Students recognize and use parent functions for linear, absolute value, quadratic, square root, and cube root to perform vertical and horizontal translations. Students understand how vertical dilations, y=k*f*(x), and horizontal dilations, y = *f*(kx), impact the graphs of the parent functions. Students make a connection between the symbolic and graphic forms of quadratic equations in the completed-square (vertex) form. Students compare two different quadratic, square root, or cube root functions represented as graphs, tables, or equations. Students write the quadratic function described verbally in a given context.	Topic C Engage Lessons 18-24	

	F-BF.B.3Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*		Students create a quadratic function from a data set based on a contextual situation, sketch its graph, and interpret both the function and the graph in context. They answer questions and make predictions related to the data, the quadratic function, and graph.		